

# Torsion in Superstring Theory

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Steven Hawking: Torsion is necessary to make superstring theory consistent.

Edward Witten: Superstring theory is a piece of 21st century physics that somehow fell into the 20th century.

Vladimir Arnold: The A-D-E graphs classify all simple mathematical objects.

<u>Coxeter-Dynkin</u>	<u>Kac-Moody</u>	<u>MacKay</u>
An: o-o-o...-o SU(n + 1)	1-1-1...-1 1	Z(n + 1)
Dn: o-o-o...-o o SO(2n)	1-2-2..2-1 1 1	Q(2n)
E6: o-o-o-o-o o	1-2-3-2-1 2 1	TD(24)
E7: o-o-o-o-o-o o	1-2-3-4-3-2-1 2	OD(48)
E8: o-o-o-o-o-o-o o	2-4-6-5-4-3-2-1 3	ID(120)

Torsion is a superfield that lives on superspace. (The antisymmetric part of the affine connection).

Superspace is a non-Riemannian manifold with both bosonic and fermionic coordinates.

Bosonic coordinates commute:

$$[x, y] = xy - yx = 0$$

Fermionic coordinates anticommute:

$$\{a, b\} = ab + ba = 0$$

The simplest version of superspace is the 8-d coset space of the 14-d graded Poincare group with respect to the Lorentz subgroup.

This is generalizable to higher dimensional superspaces, with bigger graded Lie groups.

**Example: 11-d supergravity theory with 7  
“hidden” dimensions, with E7 gauge group.**

## **The Long Strange Trip of String Theory**

### **Prehistory:**

**1914: Nordstrum- 5-d Maxwell + Newton**

**1921: Kaluza - 5-d Maxwell + Einstein**

**1926: Klein- 5-d Maxwell + Einstein +  $h/2\pi$**

### **History:**

**1969: Veneziano--beta function for 4-particle  
strong-force S-matrix**

**1970: Nambu--spining string--bosonic-26-d**

**1971: Ramond, Neveu, Schwarz--  
supersymmetry to include fermions--10-d**

1973: Brink & Nielsen--string z. p. e.

1974: Scherk & Schwarz--superstring as gravity theory; torsion.

1974: Volkov & Akulov; Salam & Strathdee--superspace as coset of graded Poincare  $\underline{P}$  group w.r.t. Lorentz group:  $\underline{P}/L = s\text{-space}$ .

1976: Wess & Zumino; Nieuwenhuizen & Freedman--supergravity.

1984: Green & Schwarz: Superstring theory as a finite, anomaly free quantum gravity theory.

1985: Gross, Harvey, Martinec, Rohm--Heterotic string-  $E_8 \times E_8$  (26-d & 10-d).

1995: Witten, Duff etc.--Membrane theory as unifier of 5 string theories.  $E_7$  (11-d).

## Tale of Two Torsions

Cartan Torsion in space of absolute Parallelism (1928):

Cartan Torsion:  $S_{jk}^i = \frac{i}{jk} - \frac{i}{kj}$

Absolute parallelism spaces (J. Wolf, 1972):  
Lie groups & Seven-sphere  $S^7$

On Lie group manifold, the Lie algebra of left-invariant vector fields provides an absolute parallelism connection.

This connection has zero curvature but non-zero torsion.

The torsion is described by the Lie algebra structure constants:  $S_{jk}^i = C_{jk}^i$

where  $[X_j, X_k] = C_{jk}^i X_i$

The non-commutativity of the Lie algebra also corresponds to the anholonomy of the group manifold, so that the structure constants also correspond to the anholonomic object:  $\overset{i}{jk} = C^i_{jk}$  .

There is a similar connection provided by the Lie algebra of right-invariant vector fields on the Lie group manifold. In this case the torsion is the negative of the structure constants:  $-\overset{i}{jk}$

There is a third basic connection on the Lie group manifold corresponding to the geodesics. This is the Levi-Civita connection  $\overset{-i}{jk}$  which is symmetric, and thus has zero torsion.

The difference between the Cartan connection and the Levi-Civita connection is the contortion tensor:  $K^i_{jk} = \overset{i}{jk} - \overset{-i}{jk}$

Poincare group:  $P = R(1,3) \times SO(1,3)$   
 [Translations  $\times$  Lorentz group].  
           4-d           +           6-d               = 10-d

Since the translations are commutative there is torsion only in the non-commutative Lorentz group part.

Graded Poincare group  $gP$ : 14-d.

Thus: cosets of  $SO(1,3)$  in  $gP$  is an 8-d superspace with both fermionic  $F$  and bosonic  $B$  parameters.  $gP/SO(1,3) = BF$ -space.

The  $F$  parameters are generated by the Majorana spinor operators (with 4 real components). The  $F$  parameters are Grassman numbers obeying the anticommutation relation:

$$\{a,b\} \quad ab + ba = 0$$

Repeated susys yield translations in

spacetime--but not in the Fermionic part of superspace.

This is a torsion that moves particles through ordinary space by “pushing” on Fermionic space.

To understand superstring theory and its membrane theory extensions use the unification structure provided by the A-D-E classifications.

Reflection groups (Coxeter, Weyl).

Hyperspace crystallographic lattices.

Particle physics eigenvalues.

Error-correcting code lattices.

Analog-to-digital quantizing lattices

Lie groups & Lie algebras.

Graded Lie groups & algebras.

Kac-Moody Lie algebras.

MacKay groups.

Singularities of differentiable maps (Arnold)  
Thom-Arnold catastrophes.

Caustics & wave fronts.

2-d Conformal field theories (non-linear  
sigma models)

Kortweg-Devries eq. & its generalizations

And much more.



