# Notes on Hyperspace

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These notes should be read as a complement (and compliment) to the article, "The Universe's Unseen Dimensions," by Nima Arkani-Hamed, Savas Dimopoulos and Georgi Dvali (*Scientific American* **283:2**, August 2000).

Their abstract reads: "The visible universe could lie on a membrane floating within a higher-dimensional space. The extra dimensions would help unify the forces of nature and could contain parallel universes."

Their central idea is that the strength of gravity is dependent on the number (and size) of the hidden dimensions. Thus in microscopic regions possibly accessible to "table top" experiments these strong gravity effects might show up.

As I will show, this idea could lead to practical methods of tapping zero-point energy. It has long been my view that zero-point energy can be properly understood only by theoretical and experimental investigation of the hidden dimensions of space-time. String theory and its membrane theory extension, called M-theory, provide a way to do this.

The generalization of string theory to membrane theory is a recent development (some say a "second revolution" in string theory) catalyzed in 1995 by the work Edward Witten, Michael Duff and others.

String theory replaces the (0-dimensional) point particles of standard quantum field theory with 1-dimensional strings, which can be open or closed in a loop. (Note that the loops can open, and close, which are topological changes.) The string idea solves at one stroke the problem that point particles have with the inverse square law. For if a force between two point particles falls off with the square of the distance between them, the force becomes infinitely strong when the point particles touch. This infinity is reflected in the major infinity problem in quantum electrodynamics, where point particles sweep out world lines in space-time and interact at point-like vertices. Replacing the point particles with loop-like strings provides world-tubes (rather than world lines) in space-time, so that interactions between strings do not occur at (point-like) vertices but at finite portions of the strings.

The strings are also vibrating; and the various harmonics of these vibrations correspond to various point particles. This correspondence is rather complicated. As the *Scientific American* article puts it: "The known fundamental particles correspond to a string that is not vibrating, much like an unbowed violin string." This state of non-vibration is the zero-mass mode of particles of various spins (0, 1/2, 1, 3/2, and 2). The higher mass modes of these particles follow a spin versus mass-squared law called a Regge trajectory. The higher vibrational states are

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separated by the huge Planck-mass-squared (i.e. $10^{38}$ $GeV^2$). The ordinary particles of the standard model can be seen only by projecting the large string theory symmetry groups (such as $E_8$ x $E_8$) down to the grand unified particle symmetry group SU(5), which in turn projects down to the standard model symmetry group U(1) x SU(2) x SU(3), for the three forces electromagnetism, weak force, and strong (color) force. The zero-mass particles pick up their masses by way of the Higgs mechanism This implies the existence of Higgs particles, which may have been detected by recent particle accelerator experiments at CERN near Geneva. Cf. Kane, 2000.

| | Spin | | | | |
|---|---|---|---|---|---|
| | 5 | | | | |
| | 4.5 | | | | |
| | 4 | | | | |
| | 3.5 | | | | f |
| | 3 | | | b | b |
| | 2.5 | | f | f | f |
| Graviton | 2 | b | b | b | b |
| Gravitino | 1.5 | f | f | f | f |
| Gauge particles | 1 | b | b | b | b |
| Quarks & Leptons | 0.5 | f | f | f | f |
| Axion & Higgs | 0 | b | b | b | b |
| Mass$^2$ in (Gev)$^2$ | | 0 | $10^{38}$ | $2(10^{38})$ | $3(10^{38})$ |

[This Regge plot is based on Figure 11.14 in "Superstrings" by Michael B. Green, *Scientific American*, September 1986; republished in Particles and Forces, R.A. Carrigan and W.P. Trower (eds.), Freeman, 1990. The b's stand for bosons (integral spin); and the f's stand for fermions (half-integral spin). The Regge trajectories are the diagonally ascending lines made by alternating b's and f's. Note that the spin/mass$^2$ slope is 1, because we are measuring mass$^2$ in basic units of Planck-mass-squared. ]

In order to make such a scheme self-consistent, space-time must be 10-dimensional, and a new symmetry must be invoked, which implies equal numbers of bosons (integral spin particles) and fermions (half-integral spin particles). This symmetry, called supersymmetry, is a transformation which changes bosons into fermions (and vice versa); and this implies that every particle must have a supersymmetry partner of opposite spin-type. For example the spin-2 graviton has a spin 3/2-gravitino partner. (Note that this is analogous to the symmetry, called charge conjugation, which transforms particles into antimatter particles, and vice versa, so that every particle has an antimatter partner.) What makes this supersymmetry doubling of particle-types palatable is threefold.

(1) Making supersymmetry a local transformation (called gauging supersymmetry) generates Einstein's general relativity equations of gravity as a subtheory. In this sense, supersymmetry implies gravity. And thus we can make the following rather striking parallelism (where Quantum theory is abbreviated as "Q. th."):

Dirac particles: (spin 1/2, fermions): Q. th. + Special Relativity → antimatter partners
Supergravity: fermions & bosons: Q. th. + General Relativity → supersymmetry partners

(2) Supersymmetry helps to solve the “hierarchy problem” of the vast energy scale difference between gravity and the other forces. The 17 parameters of the standard model must be adjusted very precisely; but supersymmetry (having no adjustable parameters), in effect stabilizes these parameters. The *Scientific American* article likens the parameter adjustment to a pencil standing on its tip; and the supersymmetry stabilization is likened to a thread holding up the pencil. Since supersymmetry entails torsion, this stabilization must be a torsion effect. (Cf. Freedman, 1981.)

(3) Supersymmetry unifies the forces precisely at a single energy scale. Since this is already partly verified experimentally, this counts as indirect evidence for supersymmetry. A direct verification will be the finding of supersymmetry partners in accelerator experiments, which will soon be attempted both at CERN near Geneva and at the Fermi accelerator near Chicago.

The remaining peculiarity of superstring theory is, of course, the hyper-dimensionality of space-time. And this is the main theme of these notes. The possibility of using knowledge of the hidden dimensions of space time to design useful devices should motivate the study of this rather abstract, but mathematically beautiful, geometry. As the motto of Plato’s Academy said:

“Let no one enter here without geometry.”

In the 19th century the mathematicians Bernhard Riemann and William Clifford independently suggested that the dynamics of the physical world was due to changes in the curvature of 3-d space.

In 1904, the mathematician Harold Hinton, working at the U.S. patent office in Washington, DC, published his book, *The Fourth Dimension,* which described 4-d space as a space potentially visible to the mind. He also provided exercises to develop this ability. His first pupils had been the two daughters of the mathematician George Boole. He married Mary Boole, but Alicia Boole was able to outperform Hinton at these exercises; and through her visualization of 4-d objects she aided in the hyperspace work of the mathematician H.S.M. Coxeter, whose graphs are the keys to hyperspace, especially as it is applied in physics. (See appendix 2.)

In 1905, the physicist Albert Einstein, working at the Swiss patent office in Bern, published his paper on special relativity. This implied a 4-d space-time, although this was not made clear until 1908 when the mathematician Hermann Minkowski gave a lecture called Space and Time,” which began with the statement:

“The views of space and time which I wish to lay before you have sprung from the soil of experimental physics, and therein lies their strength. They are radical. Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality.”

Einstein (who had been called a "lazy dog" by Minkowski at the Zurich Polytechnic Institute) was not, at first, impressed with Minkowski's union of space and time into a single 4-d space. He called this idea "superfluous sophistication." However, in order to produce a gravity theory, which entailed both special relativity and Newtonian gravity as subtheories, he found it necessary to describe gravity as the curvature of 4-d space-time. This was a fulfillment of the Riemann-Clifford idea, with the proviso that their idea of 3-d curvature be replaced by curvature of Minkowski's 4-d space-time.

Thus Einstein's 1915 theory of gravity (called general relativity) was the first successful hyperspace theory in physics. The next challenge was to bring electromagnetism into the hyperspace picture. In the 1920's Theodore Kaluza and Oskar Klein accomplished this by introducing a 5th dimension to space-time. Klein described this 5th dimension as a tiny loop, whose size is a Planck length ($10^{-35}$ meter), one such loop at every point of space-time. We can visualize this by projecting 4-d space-time to 1-d, the 5-d space would be projected to a 2-d space looking like a very long garden hose with a $10^{-35}$ meter cross section. This tiny cross section would account for the invisibility of the 5th dimension to us macroscopic beings who seem to live along the length of the garden hose projection. Klein used this 5-d theory to derive his relativistic version of the Schroedinger equation of quantum mechanics. This equation is called the Klein-Gordon equation; and, in fact, Klein's motivation in developing the 5-d theory was to find a way to understand quantum theory from the geometrical point of view of general relativity. This was a prelude to the much later development of superstring theory. (Cf. *Modern Kaluza-Klein Theories,* Applequist, Chodos, & Freund, 1987.)

The Kaluza-Klein theory was forgotten in the avalanche of work on the exciting new discoveries in the 1930s in nuclear physics. Here were found two nuclear forces: the strong (which holds the nucleus together against the electrical repulsion of the protons); and the weak (which is mainly evident in a certain change of the nucleus, called "beta decay" that changes the type of atom.)

In 1974 (after a 50-year hibernation) the Kaluza-Klein theory was revived by Joel Scherk and John Schwarz, who proposed string theory as a quantum gravity theory, capable of unifying all the forces. This idea evolved into 11-d supergravity, which is a particle theory rather than a string theory. By 1984 when supergravity ran into trouble with infinities (the technical term is "nonrenormalizability"), John Schwarz and Michael Green created a 10-d string theory obeying10-d supersymmetry, which was both finite and anomaly free. This means that the infinities of supergravity do not show up, because the interactions are string interactions; and also there would be no negative energy states and no unphysical chiral states (i.e., states of only one handedness), provided that the theory obeyed a gauge symmetry described by the 496-dimensional Lie groups SO(32) or $E_8$ x $E_8$. This was a strong proposal for a quantum gravity theory with the ability to include all the forces in a unified string theory. This result created a tidal wave of activity, which seems to be increasing every year. The end is not in sight. There is much more to be discovered in this theory, which Witten describes as "a piece of 21st century physics that somehow fell into the 20th century." [For a brief description of group theory, see Appendix 1.]

It soon became clear that there were five competing superstring theories, each of which had a "low energy" point-particle subtheory. These subtheories are considered as approximations, because the phrase "low energy" means that these point particle field theories break down when the point particles get too close to each other and the energy of the interaction increases without bound (i.e. goes to infinity).

The most interesting of these string theories is the $E_8$ x $E_8$ theory, which was developed into a heterotic string theory by the Princeton String Quartet (whose members were D.J. Gross, J.A. Harvey, E. Martinec, and R. Rohm). The word "heterotic" means that this string theory is a subtle interweaving of the original (1970) bosonic string theory (with 26 space-time dimensions) and superstring theory (with 10 space-time dimensions). Notice that 26 - 10 is 16. And also notice that 8 + 8 is 16. These simple numbers are the keys to this scheme. $E_8$ x $E_8$ is a 496 dimensional algebra, with a 16 dimensional commutative subalgebra (called the Cartan subalgebra). The dual space to this 16-d subalgebra is a 16-d space in which 480 vectors (called roots) carry the charges (16 for each vector) that identify each of the 480 non-commutative generators of the $E_8$ x $E_8$ symmetry group. There are 16 commutative generators provided by the 16 dimensional Cartan subalgebra. Thus all 496 generators (16 + 480) of $E_8$ x $E_8$ are accounted for. The geometry of the 16-dimensional space is best described by the Coxeter graph (or Dynkin diagram) for $E_8$ x $E_8$, and notice that we have previously mentioned H.S.M. Coxeter. [See also the table of A-D-E Coxeter graphs in Appendix 2.]

```
*--*--*--*--*--*--*  *--*--*--*--*--*--*
      |                        |
      *                        *
```

Each node (*) indicates a basic mirror (a 15-d hyperplane) which cuts the 16-d space so that all the space on one side of the mirror can be flipped to the other side (and vice versa). Thus there is a positive mirror vector (and a negative mirror vector behind the mirror) for each basic mirror. These mirror vectors are called roots; and the positive roots are the basic roots. The line (--) between any two nodes indicates a 120° angle between two basic roots. The lack of a line between any two nodes indicates a 90° angle between two basic roots. Note that we have two copies of the $E_8$ graph; and that these two graphs are not connected by a line, so that there is a 90° angle between the two "connecting" roots.

Coxeter calls this set of mirrors a kaleidoscope by analogy with the toy kaleidoscope. Just as the mirrors in such a toy kaleidoscope (with two mirrors set at 60°) generate a new virtual mirror, so the 16 basic mirrors of the $E_8$ x $E_8$ kaleidoscope generate 224 virtual mirrors providing a total of 240 (= 16 + 224) mirrors and thus a total of 480 roots, as mentioned above. Since each root carries the charges which correspond to a particle type, the change of one particle into another is accomplished by a reflection, or a series of reflections across the 16 basic mirrors.

Since the 16 basic mirrors generate all the mirrors, and in so doing reflect the roots into each other, the 16 basic reflections generate a finite reflection group, called the Coxeter group (or Weyl group) of the Coxeter graph. In particle physics, these Coxeter reflections correspond to the action of force particles on matter particles, thus changing one type of particle into another.

In more detail: the basic rule of particle interaction is that matter particles interact by exchanging force particles; and force particles interact by exchanging force particles. Here we can see that this rule is accomplished by the mirrors (corresponding to roots, which carry force particle charges). Ordinarily, the matter particles correspond to "weight" vectors carrying matter particle charges. In the case of $E_8$ (and thus $E_8$ x $E_8$), however, there is no distinction between force and matter particles. This is a consequence of the self-duality of the $E_8$ lattice. Ordinarily, the weight lattice (carrying matter-type charges) is dual to the root lattice (carrying force-type charges). These lattice differences separate out when the $E_8$ lattice is projected down to lower dimensional lattices; and these projections are necessary to make contact with ordinary particle physics, as we will see.

The $E_8$ x $E_8$ Lie algebra guarantees anomaly cancellation in the heterotic string theory. And this is precisely because the $E_8$ x $E_8$ root lattice is an "even, self-dual" lattice. Thus this highly symmetric lattice provides for cancellations of anomalies that would otherwise spoil the self-consistency of the string theory.

An apparently little known fact is that every even, self-dual lattice generates an even, self-dual error-correcting code, (cf. Conway and Sloane, p. 186) so that it is really the error-correcting code properties that are canceling the anomalies. It so happens that the $E_8$ x $E_8$ root lattice generates two copies of the Hamming-8 error-correcting code, which is a code of word-length 8, with 4 message carrying (binary) digits and 4 error correcting digits. So the $E_8$ x $E_8$ code has word-length 16, with 8 message carrying digits.

However, we still have to worry about the 26-dimensional space-time with 22 hidden dimensions. Actually the basic object in string theory is the 2-d world sheet swept out by the interacting strings (analogous to the world lines swept out by point particles in the Feynman diagrams of quantum field theory). We can picture this world sheet as vibrating in the hyperspace. Since the vibrations must be transverse to the world sheet, there are 24 vibrational degrees of freedom in the 26 dimensions of heterotic string theory.

We can describe this 24 dimensional (vibrational) space as the space of a very powerful error-correcting code, the Golay-24 code. This code has 12 message carrying digits and 12 error-correcting digits, which can correct three errors anywhere along the 24-bit code word. The Golay-24 code is an even, self-dual error correcting code. The importance of the Hamming-8 code ( $E_8$) is described by Gleason's theorem stating that every even, self-dual error correcting code can be generated by two codes, the Hamming-8 code and the simple square-lattice code {00,11}. (Cf. Conway and Sloane, p. 186; Goddard and Olive.) It is significant that this most basic theorem of algebraic coding theory should play a role in unified string theory.

The way in which the 24-d vibrational space (of 26-d bosonic string theory) relates to the 8-d vibrational space (of 10-d superstring theory) is that the Golay-24 code can be derived from 3-copies of the Hamming-8 code, i.e. from the Coxeter graph for $E_8$ x $E_8$ x $E_8$ .

Strangely enough the vibrational dimensionality of 24 can be derived from a solution to a 24th degree polynomial equation called the Ramanujan function (after the mathematical wizard from India). A similar solution to an 8th degree polynomial accounts for the vibrational dimension 8. Michio Kaku (1994) describes this in the book Hyperspace. The dimensions can also be derived by a calculation which assumes that the vibrations of the strings are zero-point energy fluctuations (Brink & Nielson, 1973). Thus there must be a deep relationship between quantum vacuum fluctuations, the Ramanujan modular function, and error-correcting codes. This is an example of vastly different mathematical objects being different windows into some deeply lying structure. As a hint of things to come, I will mention that the Coxeter graphs are the key to the relationships between many different mathematical objects. (See Appendix 2.)

In order to show that string theory entails ordinary particle physics, it is customary to invoke the hierarchy of Coxeter graphs (by successively removing nodes):

```
E8                  → E6            → D5          → A4

*¯¯*¯¯*¯¯*¯¯*¯¯*¯¯*  → *¯¯*¯¯*¯¯*¯¯*  → *¯¯*¯¯*¯¯*  → *¯¯*¯¯*¯¯*
     |                       |              |
     *                       *              *
```

where $D_5$ corresponds to the Lie group SO(10), and $A_4$ corresponds to SU(5). And SU(5) can be recognized as the Georgi-Glashow (1974) grand unified theory (GUT) of the three gauge forces electromagnetism, weak force, and strong color force, with gauge groups U(1) x SU(2) x SU(3) as a maximal subgroup of SU(5). The four nodes in the Coxeter graph $A_4$ (for the SU(5) GUT theory), correspond to the 4 charge dimension necessary to describe the gauge interactions: 1 dimension for electric charge, 1 dimension for weak charge, and 2 dimensions for strong color charge.

Now since the $A_4$ graph is a subgraph of the $E_8$ graph, we can see that from the point of view of an error-correcting code, the $A_4$ part of the $E_8$ graph is the message carrying part of the code-word (describing the charges of GUT theory), while the remaining 4 nodes correspond to the error-correcting digits. The message carried by the code is, presumably, just the world as potentially experienceable by conscious entities. Since error-correcting codes are an integral part of information theory, it is plausible that the mind-like aspect of reality intertwines with the matter-like aspect of aspect of reality by way of these codes. (Cf. Sirag 1993, 1996.)

There is another peculiarity in the above hierarchy of graphs (which is standard in string theory). Notice that we removed two nodes to get from $E_8$ to $E_6$. We skipped over $E_7$. This was a deliberate omission, because $E_7$ is incompatible in a certain sense with "chiral fields." This means that the weak charge, which distinguishes, between right and left-handedness could not properly be modeled.

[Personal Note: $E_7$ has long been my favorite Lie algebra because of its correspondence to the Octahedral group. For details see my paper "Consciousness: a Hyperspace View," in The *Roots of Consciousness* by Jeffrey Mishlove (1993). In 1986 I showed my paper "An $E_7$ unification scheme via the Octahedral Double group," to Abdus Salam. He said immediately, "I thought $E_7$ was dead." And I said, "Well, I'm going to resurrect it." In 1989 I gave a copy of a later version of this paper to Edward Witten. By 1995 Witten succeeded in resurrecting $E_7$ via membrane theory. Perhaps I had a little something to do with this.]

The extension of string theory to membrane theory (called M-theory for “membrane, magic or mystery” according to Witten), changes the picture radically. M-theory itself is an 11-dimensional theory with 11-d supergravity as its low energy approximation. The 7 hidden dimensions of this 11-d supergravity theory is the 7-d torus, which is the Cartan subgroup of $E_7$.

The $E_7$ Coxeter graph is:

```
*¯*¯*¯*¯*¯*        (with the E7 extended graph)      1-2-3-4-3-2-1
    |                                                      |
    *                                                      2
```

Note that every Coxeter graph has an extended form, which has many uses. The numbers on the nodes of the extended graph I call balance numbers, because they indicate the relative lengths of the roots necessary to have the roots in balance, i.e. as vectors they sum to zero. The sum of the balance numbers is called the Coxeter number. Thus the $E_7$ Coxeter number is 18. From the Coxeter number K of a Coxeter graph we can derive the dimensionality D of the Lie algebra (and the Lie group) by the formula:

$D = RK + R$: D = Non-commutaive Lie algebra dim. + Commutative dim.

where R is the rank (the number of nodes) of the graph. RK is the number of (non-zero) roots, while R is the number of zero roots corresponding to the dimensionality of the maximal commutative subalgebra. At the group level this is the Cartan subgroup which is a maximal torus of dimension R. Since the torus is commutative, its Lie algebra (the Cartan subalgebra) is commutative. The non-zero roots correspond to the non-commutative part of the Lie algebra (and its Lie group) and thus the total dimensionality of the Lie algebra (and its Lie group) is D.

[Note that the non-commutative part of a Lie algebra corresponds to torsion of the Einstein-Cartan unified field theory (Cf. M.A. Akivis & B.A. Rosenfeld, Elie Cartan (1869-1951), American Mathematical Society, 1993); and Elie Cartan & Albert Einstein, Letters on Absolute Parallelism, 1929-1932, edited by Robert Debever, Princeton, 1979).]

In the case of $E_7$, we have: $D = 133 = 7(18) + 7$. Thus there are 126 non-commutative dimensions, and 7 commutative dimensions in the Lie algebra (and the Lie group). And thus there are 126 non-zero roots, corresponding to 63 mirrors. These 126 roots are the kissing points on a 6-d sphere $S^6$, so that 126 spheres (each an $S^6$) pack most efficiently around a central $S^6$ in 7-d space. This is the optimal solution to the sphere packing problem for a 7-d space. These roots also generate an error-correcting code, in fact the Hamming-7 code consisting of $2^7 = 128$ (binary) code words of length 7, with 4 message-carrying digits, and three error-correcting digits. This code is closely related to the Hamming-8 code of $E_8$, for the Hamming-7 code becomes the Hamming-8 code with the addition of a single parity-check digit. Both of these codes correct for one error (a change of 0 to 1, or 1 to 0) in any digit in a code word of length 7 (or 8).

Since, the Hamming-8 code generates the lattice that provides anomaly cancellation in the $E_8$ x $E_8$ heterotic string theory, I claim that anomalies should also be canceled in the $E_7$ supergravity theory (and M-theory). This is because it is really the error-correcting code that cancels the anomalies; and the Hamming-7 code corrects the same number of errors per codeword as the Hamming-8 code. So it is not so surprising that $E_7$ in the 11-d theory should be restored to a place of honor above the 5 competing superstring theories. As the old saying goes: “The stone that the builders rejected has been made the head of the corner.”

There is a dimensional hierarchy of theories (cf. Kaku, 1999, p. 462), where 11-d, 10-d, and 9-d, refer to uncompactified space-time dimensions):

```
11-d                M-theory
       (open str. ) /          \   (closed strings)
10-d     IIB     IIA        E8 x E8        SO(32)  <-> I  (SO(32), open & closed strings )
            \      /              \             /
 9-d          II       <->     Heterotic (compactified to 9-d)
```

The double arrow symbol ←→ indicates a duality, which is somewhat analogous to the duality between electricity and magnetism (if we assume the existence of magnetic monopoles), for then there is a duality between the strength of the electrical charge and the monopole charge -- i.e., a duality between the pure numbers 1/137 for electric charge, and 137 for magnetic charge.

Each of the five 10-d superstring theories has a (low-energy) subtheory: a point-particle supergravity theory, in which there is a superspace with N supersymmetries, that determines the dimensionality of the non-commutative part of superspace. The maximum number of possible supersymmetries is N = 8. In this theory, the numbers of various particles are (along with the relevant symmetry groups):

| | | |
|---|---|---|
| 1 spin 2 | -- graviton | |
| 8 spin 3/2 | -- gravitino | --SO(8) (fundamental representation) |
| 28 spin 1 | -- gauge particles | --SO(8) (adjoint representation |
| 56 spin 1/2 | -- matter particles | --$E_7$ (fundamental representation) |
| 70 spin 0 | -- scalar particles | --$E_7$/SU(8) -- [133 - 63 = 70 ] |

These correspondences suggest that the 11-d supergravity theory starts with N = 1, E7 as global symmetry group, and the subgroup SU(8) as local (gauge) symmetry group. But when the 11-d supergravity is compactified on the 7-d torus $T^7$ (i.e., the Cartan subgroup of $E_7$), so that only the ordinary 4 dimensions of space-time are left unhidden, then N increases to 8 and the gauge group reduces to SO(8).

Notice that in this transformation the superspace dimensionality remains as 12; that is, 11 bosonic dimensions + 1 fermionic dimension becomes 4 bosonic dimensions + 8 fermionic dimensions. However, we must remember that in the latter case, the 11 non-compact dimensions have changed to an 11-d space in which 7 are compact (hidden) dimensions. Moreover, this compactification is on the 7-d torus, which is the Cartan subgroup of the $E_7$ Lie group.

However, having done all this, M-theory reveals that 11-d supergravity is merely the point particle approximation to 11-d M-theory theory. Remember that point particle interactions lead to infinities that spoil the theory, so that point particle theory must be regarded as a subtheory (approximation) to some more precise theory. M-theory is not a point-particle theory, but neither is it a string theory in its 11-d form. Rather it contains 2-branes (ordinary membranes) and 5-branes (5-d membranes). This is true for the uncompactified theory -- i.e. no space-time dimensions hidden by compactifying into a tiny space, such as a circle, torus, Calabi-Yao space, orbifold, and so on.  The membranes that are possible at various levels of compactification have been calculated  (cf. Kaku, 1999, p. 514) and are indicated by the Xs in the following table, called a "brane-scan" (where a p-brane is a membrane of dimension p):

p-brane dimensions:

| | | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|---|
| No. of uncompactified space-time dimensions: | 11 | | | x | | | * |
| | 10 | | x | | | | x |
| | 9 | x | | | | x | |
| | 8 | | | | x | | |
| | 7 | | | x | | | |
| | 6 | | x | | x | | |
| | 5 | x | | x | | | |
| (ordinary space-time) | 4 | | x | x | | | |
| | 3 | x | x | | | | |
| | 2 | x | | | | | |

Note: p-branes sweep out (p+1)-spaces

0-brane = point particle → world line
1-brane = string → world sheet
2-brane = membrane → world volume
3-brane = 3-space →  world 4-volume
4-brane = 4-space →  world 5-volume
5-brane = 5-space →  world 6-volume

Notice that for 11-d M-theory, the 5-brane is indicated by an asterisk, which indicates that this membrane is not fundamental, but arises by duality with the 2-brane. Notice also that it does not fit the overall pattern of diagonal x's, so the 5-brane is an odd man out. There would be other duality invoked membranes, that are not shown.

Beside these "ordinary" p-branes, there are other types. Especially important are a type of p-brane called a D-brane, where the D stands for Dirichlet, after the mathematician Peter Gustav Lejeune Dirichlet (1805-1859). The D refers to a certain boundary condition called the Dirichlet boundary condition, which implies that open strings can attach to a D-brane.  In M-theory,  the D-brane is like a wall (not necessarily flat) to which open strings are attached at their ends. There are D-branes of different dimensions, just like the ordinary p-branes described above.  For example, Type IIA superstring theory (with 10 uncompactified dimensions) contains even dimensional D-branes; while Type IIB superstring theory (with 10 uncompactified dimensions) contains odd dimensional D-branes.  There is a duality between the even and odd dimensional D-branes by way of the compactification of one dimension, so that the two Type II theories unify at the level of 9 uncompactified dimensions. (This is just a small part of the picture, as is clear by looking at the diagram of the hierarchy of theories unified by M –theory.)

The authors of the *Scientific American* article speculate that the entire 3-d space of the universe (as ordinarily understood) could be a 3-dimensional D-brane, embedded in a much higher dimensional space. Such a model is possible in the 10-d uncompactified (Type IIB) superstring space. In this scenario, the fact that gravity (unlike the other forces) can escape the universe D-brane means that in high-energy accelerator experiments the escaping gravity may show up as missing energy. Moreover the gravitational force constant (Newton's G) may not be constant at tiny distances. The idea is that the extra compact dimensions may hide the true strength of the gravitational force. At the tiny distances approaching the compact dimensions, Newton's inverse-square law of gravity would be replaced as follows (for spatial dimensions):

| | | | | | |
|---|---|---|---|---|---|
| 3-d: | no hidden dimensions | | $1/R^2$ | in | $F = G(m_1 \times m_2)(1/R^2)$ |
| 4-d: | one | " " | $1/R^3$ | replaces | $1/R^2$ |
| 5-d: | two | " " | $1/R^4$ | " | |
| 6-d: | three | " " | $1/R^5$ | " | |

and so on. The rule is that for n hidden dimensions the gravitational force falls off with the inverse (n + 2 ) power of the distance R.

This implies that as we look at smaller and smaller distances (by banging protons together in particle accelerators) the force of gravity should look stronger and stronger. How much stronger depends on the number of hidden dimensions (and how big they are). There may be enough hidden dimensions to unify the all the forces (including gravity) at an energy level of around 1 TeV ($10^{12}$ eV), corresponding to around $10^{-19}$ meters. This would be a solution to the hierarchy problem of the vast difference in energy scale between the three standard gauge forces and gravity. This is already partly solved by supersymmetry (as mentioned previously); but this new idea would be a more definitive solution--if it were the right solution!

In this greatly revised picture of the unification energy level, the exotic new particles predicted by the string theories should show up at the new proton collider called LHC (Large Hadron Collider) now being built at CERN near Geneva. It is scheduled to be in operation by 2005.

Given the revised unification scale, an even more exotic particle could show up at the LHC -- a micro black hole. These black holes would be created by the high-energy collision of protons. According to the *Scientific American* article, the size of these black holes would be around $10^{-19}$ meters; but because they would evaporate in $10^{-27}$ seconds, there would be no danger.

If two or more of the hidden dimensions are large enough, say a micron ($10^{-6}$ meters) or even a millimeter it would be possible to see effects with "table top"-sized physics experiments. Such an experiment is described very briefly in the in the *Scientific American* article. More details are available at the website: http://mist.npl.washington.edu/eotwash/

It has occurred to me that various researchers, who have claimed to be tapping zero-point energy via piezoelectric devices, may be tapping into the hidden dimensions of string theory (in its membrane theory extension). I say this for two reasons:

(1) The membrane theory provides a way to move the strength of gravity close to the electro-weak unification scale. This would be the case if there are enough hidden dimensions and/or they are big enough.

(2) The vibration of the strings is due to zero-point energy fluctuations. In fact (as previously mentioned) the dimensionality of string theory, 26-d for bosonic strings and 10-d for superstrings can be derived by a zero-point energy calculation on the strings. Brink and Nielsen published this result in 1973.

Thus the zero-point fluctuations of the strings, if the strings are big enough, could be transmitting effects by way of strong gravity at the microscopic scale, which might be available to devices utilizing piezoelectric vibrations as a transducer.

In order to properly utilize such devices, it will be necessary to clarify the physics of hyperspace. The work on string theory and its M-theory generalization is a rapidly developing approach to this physics. A vital mathematical tool in this work is the set of Lie algebras classified by the A-D-E Coxeter graphs. (See Appendix 2.)

I believe that the Coxeter graphs (also called Dynkin diagrams) for the A-D-E subset of graphs (which according to V.I. Arnold classify all "simple" mathematical objects) is a set of Platonic forms for reality. These A-D-E graphs characterize a vast mathematical object underlying many different types of mathematical-physics object. This vast underlying object must be reality giving rise to all its forms.

Arnold hints this at in his book, *Catastrophe Theory:*

"At first glance, functions, quivers, caustics, wave fronts and regular polyhedra have no connection with each other. But in fact, corresponding objects bear the same label not just by chance: for example, from the icosahedron one can construct the function $x^2 + y^3 + z^5$, and from it the diagram $E_8$ and also the caustic and wave front of the same name.

"To easily checked properties of one of a set of associated objects correspond properties of the others which need not be evident at all. Thus the relations between all the A, D, E classifications can be used for the simultaneous study of all simple objects, in spite of the fact that the origin of many of these relations (for example, of the connections between functions and quivers) remains an unexplained manifestation of the mysterious unity of all things."

## Appendix 1: Group Theory, the bare bones

A **group** is a set of elements obeying some of the rules that numbers obey so that for a group G, and elements a, b, c, etc.:

**Closure:** if a and b are elements of G, then c = ab is an element of G

**Associativity:** a(bc) = (ab)c
**Identity element:** ae = ea = a, for the single identity element e of G
**Inverses:** every element a has an inverse $a^{-1}$ so that $aa^{-1} = a^{-1}a = e$

A **commutative group** obeys an extra rule, **Commutativity:** ab = ba
A **semi-group** omits the inverses rule: some elements have no inverse

A **finite group** has a finite number of elements. Examples: Coxeter (reflection) groups; permutation groups; finite subgroups of Lie groups; where SU(2) is the most important example of a Lie groups whose finite subgroups I call McKay groups (cf. Appendix 2).

A **discrete group** has either a finite or countable infinity number of elements. Examples of **infinite discrete groups**: integers, Coxeter lattices in Lie algebras generated by extended Coxeter graphs.

A **Lie group** is a smooth space (a differentiable manifold), in which each point of the space is an element of the group. Note that a Lie group thus has a **continuous infinity** of elements.

A **Lie algebra** is the set of vector fields that live on the Lie group manifold such that they obey a symmetry condition called "left-invariance" which allows all the vectors in a single vector field to be identified with a single vector at the identity element of the Lie group. This implies that the tangent-vector plane at the identity element is another way of defining the Lie algebra, and thus that the Lie group and its Lie algebra are different spaces of the same dimensionality. The Lie algebra is not an associative algebra.

A **linear associative algebra** is a vector space that obeys rules similar to those of ordinary algebra (of real or complex numbers). There is an additive group (with an identity element called 0), and a multiplicative semi-group (with an identity element called 1). Any such algebra can be made into a **Lie algebra** by imposing the **Lie product** ([a,b] = ab -ba) on elements of the underlying associative algebra.

A **representation** of a group (or an algebra) is a correspondence between the elements of the group (or algebra) and transformations of a **vector space**. This correspondence is realized by sets of **square matrices**, so that an n-dimensional representation transforms an n-dimensional vector space via n x n matrices. Transformations act on the vectors of the vector space by stretching (or shrinking) them, rotating them, or even some combination of stretching ( or shrinking) and rotation.

A **faithful representation** identifies each element with a unique matrix. There are two basic faithful representations: the **fundamental** (or minimal faithful) representation and the **regular** (or **adjoint**) representation. For a finite group with n elements, the regular representation utilizes n x n matrices. For a Lie group (or Lie algebra) of dimension n, the adjoint representation utilizes n x n matrices. For example (cf. p. 9 above):

Finite group: OD: 2-d (complex) fundamental, and 48-d regular representations.
Lie group: $E_7$: 56-d (complex) fundamental, and 133-d adjoint representations.

Finite group: ID: 2-d (complex) fundamental, and 120-d regular representations.
Lie group: $E_8$: 248-d (complex) fundamental, and 248-d regular representations.
(Note: this is a special case, in which the $E_8$ lattice is self dual.)

**Appendix 2: The A-D-E Coxeter Graphs (Dynkin Diagrams)**

**(ADEX-theory: the study of all the A-D-E-classified objects )**

| Label: | Coxeter graph: | Compact Lie group: | Total number of mirrors: | McKay subgroup of SU(2): # of elements: |
|---|---|---|---|---|
| $A_n$: | *--*- .... -* | SU(n +1) | $(n^2 + n)/2$ | cyclic: $Z_n$ (n + 1) |
| $D_n$: | *--*--*-....-*<br>\|<br>* | SO(2n) | $n^2 - n$ | dihedral double:<br>Ddn: (2n) |
| $E_6$: | *--*--*--*--*<br>\|<br>* | E(27) | 36 | tetrahedral double:<br>TD: (24) |
| $E_7$: | *--*--*--*--*--*<br>\|<br>* | E(56) | 63 | octahedral double:<br>OD: (48) |
| $E_8$: | *--*--*--*--*--*--*<br>\|<br>* | E(248) | 120 | icosahedral double:<br>ID: (120) |

Note: These graphs classify many mathematical objects of great importance in physics (especially string theory and M-theory). Thus they provide a way to transform from one type of object to another. They include:

Lie algebras (and Lie groups), (Gilmore 1974, Georgi 1980)
Kac-Moody (infinite-d) algebras (Kaku 1999, Kac, Julia)
Coxeter (reflection) groups (also called Weyl groups). (Coxeter)
McKay groups (finite subgroups of SU(2)). (McKay)
Hyperspace crystallography (Coxeter, Conway & Sloane)
Sphere-packing lattices (root lattices); error-correcting codes (Conway & Sloane)
Quantizing lattices (weight lattices); analog-to-digital transforms (Conway & Sloane)

Conformal field theories (which live on the 2-d string world-sheet) (Kaku, 1999)
Gravitational instantons (providing a link with Penrose twistors) (Kronheimer 1989, ’90)
Arnold-Thom catastrophes (Thom: Arnold 1981, 1986; Gilmore 1981)
Singularities of differentiable maps (Arnold 1981, 1986)
Heisenberg algebras (in various hyperspaces) (Kostant)
Korweg de Vries hierarchy of non-linear equations (Julia)
Generalized Braid groups (related to knots and links0 (Kauffman)

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