Review

At noncritical points $\nabla Q^* \neq 0$

$Q^* \doteq y_1 + \text{constant}$

$y_j = y_j(x_1, x_2, \ldots x_n)$

$j = 1,2,\ldots n$

At a Morse critical point $\nabla Q^* = 0$,

$\mathrm{Det}Q^*_{ij} \neq 0$

The stability matrix Q^*_{ij} is

$$Q^*_{ij} = \frac{\partial^2}{\partial x_i \partial x_j} Q^*$$

$$Q^* \doteq \sum_{i=1}^{n} \lambda_i y_i^2$$

The matrix Q^*_{ij} has eigenvalues λ_i that can be normalized to -1 p times and to +1 n-p = q times suggesting the Lorentzian metric group O(p,q) in the neighborhood of a Morse critical point's tangent space to the curved mental landscape manifold in configuration space.

For non-Morse critical points in which

$\mathrm{Det}Q^*_{ij} = 0$

$$Q^* \doteq f_{NM}(y_1(x, X), \ldots, y_\ell(x, X))$$

$$+ \sum_{j=\ell+1}^{n} \lambda_j(X)(y_j(x))^2$$

where the set X of n control parameters are the actual positions (AKA "source points") of the n material dynamical degrees of freedom. x is the general "field point" in configuration space.

Note that the use of $\mathrm{Cat}(\ell,n)$ in Lecture 2 only works when n is less than or equal to 5. Thus, it is not useful. We can use $\mathrm{CG}(\ell)$ in which $\mathrm{Pert}(\ell,n) = 0$ for all n.