From St Bernard to String Theory
And the Mystery of 133

Saul-Paul Sirag, April 11, 2000

According to Agent X, as quoted in an email from Jack Sarfatti:

"By 1127 the Knights Templar had found the treasure in Solomon's stables below the Temple and St Bernard of Clairvaux convened a great council at Troyes in 1128 under the protection of the Count of Champagne."

Correction (to Agent X): Solomon never had stables beneath the Temple. The Knights Templar did, however, stable their horses beneath the temple compound, which by then was an Islamic compound, with the Dome of the Rock as centerpiece. Solomon's stables for chariot horses were at the fortress he had built at Meggido--a strategic location near the valley where many battles have been fought. The book of Revelation locates the battle of Armageddon at this same valley.

It should also be noted that it was St Bernard, who set up the initiation hymn for the Knights Templar as Psalm 133. And to this day, the Bible is open to Psalm 133 during the First degree initiation rite of Freemasonry.

This number 133 gets around. Jack hapens to live in North Beach in zip code 94133. And 133 is the dimensionality of the E7 Lie group, which I have been studying for a long time now, since the E7 McKay group is OD, the double octahedral group. One way to see the connection between OD and E7 is to write down the extended E7 Coxeter graph:

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1--2--3--4--3--2--1
         |
         2
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This graph represents (among many other things) 8 vectors in balance in the 7-d reflection space of E7. The balance numbers are the lengths of these vectors, which are set at 120 degrees to each other whenever a line in the graph connects two balance numbers.

The sum of the E7 balance numbers is 18, which is called the E7 Coxeter number.

If we multiply 18 by 7, we get 126, which is the number of (6-d) spheres the pack around a central sphere in 7-d space. The 126 spheres touch that central sphere in 126 kissing points; so 126 is called the E7 kissing number. The kissing points can be regarded as vectors in the reflection space. These vectors are called roots, and they carry (as components) the eigenvalues of force-type particles (called bosons). Thus each root corresponds to an eigenvector, and the 126 eigenvectors form a basis for 126 non-commutative dimensions of the E7 Lie algebra (which is the set of left-invariant vector fields living on the manifold of the E7 Lie group. There remain 7 commutative dimensions, which have as basis the 7 roots described by the E7 (non-extended) Coxeter graph:

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1--1--1--1--1--1
      |
      1
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where the 1s indicate the uniform lenghts of the basis roots. Note that in the extended graph we added a node and lengthened the roots by factors indicated by the balance numbers. Each of the basis roots is attached to a 6-d hyperplane, called a mirror. And (analogous to a kaleidoscope) these basic mirrors generate 56 new mirrors, so that there are a total of 63 mirrors. Attached to each mirror is a root on one side, and a negative of this root on the other side. Thus there is a total of 126 roots. This generates the total dimensionality of the Lie group (and the Lie algebra) as 7 + 126 = 133.

Now we can do something else with the balance numbers of the extended E7 Coxeter graph. We can multiply all the balance numbers together to get 288, and if we multiply this number by 5040 which is 7! (that is 7 x 6 x 5 x 4 x 3 x 2 x 1, for the permutational symmetry of the 7 basic mirrors), we get 1451520, and if we multiply this number by 2 (for the bilateral symmetry of the Coxeter graph itself), we get 2903040, which is the number of elements in the E7 reflection group (also called the Coxeter group or the Weyl group). Each element is a combination of the basic refletions afforded by the 7 basic mirrors. These 2903040 elements also correspond to the number of chambers generated by the 7 basic mirrors forming one fundamental chamber--just as the 6 elements of the 2-d reflection group of the ordinary kaleidoscope generate 6 chambers from one chamber (containing the pieces of colored glass) which give the snowflake patterns.

Adding an extra mirror, indicated by the extended graph, closes of the fundamental chamber so that an infinity of chambers is generated. This corresponds to an infinite element reflection group and also generates an infinite dimensional Lie algebra called the E7 Kac-Moody algebra, of which the 133 dimensional Lie algebra is the maximal dimensional sub-algebra. (Note that adding a third mirror in an ordinary kaleidoscope does something similar, and corresponds to the A2 Kac-Moody algebra.)

A third thing we can do with the E7 balance numbers is to add the squares of these numbers: 1 + 4 + 9 + 16 + 9 + 4 + 1 + 4 = 48. And this number 48 gives us the number of elements in the E7 McKay group. I call this a McKay group because John McKay in 1979 proved that the 8-d Cartan subalgebra of the E7 (infinite dimensional) Kac-Moody algebra is equivalent to the 8-d center of the 48 dimensional OD group algebra. And this formulation is general for all of the A-D-E Lie algebras. For each of these Lie algebras there is a McKay group; and the McKay groups are the finite subgroups of one particular Lie group called SU(2)--defined as the set of all special unitary 2-by-2 matrices. Geometrically SU(2) is the 3-d hypersphere S^3--the set of unit-length vectors in 4-d space. The McKay groups are sets of very symmetrical points on S^3.

For example, the E6 McKay group is TD, the double tetrahedral group of 24 elements. These elements are 24 points which are the vertices of theexquisitely symmetric object, which Coxeter calls the 24-cell. (This is a self-dual 4-d polytope, which has both 24 vertices and 24 hyper-faces, or cells, each of which is an octahedron.)

The E7 McKay group is OD, the double octahedral group of 48 elements. Since TD is a subgroup of OD, it is not surprising that the 48 elements of OD are 48 points on S^3 which are generated by two 24-cells reciprocal to each other.

The E8 McKay group is ID, the double Icosahedral group of 120 elements. And this corresonds to 120 points on S^3, which form the vertices of a 600 cell, each cell being a tetrahedron. Moreover, since TD is a subgroup of ID, these 120 points correspond to 5 copies of the 24-cell.

These three are the only E-type graph structures, and they entail all the 5 platonic solids, because the octhedron is dual to the cube, while the icosahedron is dual to the dodecahedron. So Plato's forms have come back in a big way in the 20th century. In fact, the Russian mathemetician V.I. Arnold calls the study of the A-D-E graphs platonics. Since 1965, he has inaugurated a program to use these A-D-E graphs to classify all "simple" mathematical objects. A few of these A-D-E classified objects are: Coxeter's reflection groups, Lie algebras (and groups), Thom catastrophe structures, singularities of differentiable maps, caustics, hyperspace crystallographic structures, 2-d conformal field theories, and gravitational instantons. Many of these objects are key structures in unified field theory in the form of string theory. My belief is that the entire A-D-E hierachy in all its classifications is entailed in string theory (and its extensions to M-theory). String theorists have been looking for an underlying principle, analogous to Einstein's equivalence principle for general relativity. My proposal for this principle is that reality is the structure underlying the entire A-D-E hierarchy in all its guises. The particle physicist, Leon Lederman, the book, "The God particle," said that the final equations we are looking for should be so simple they could be printed on a t-shirt. Rather than equations, I propose the A-D-E diagrams:

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An:  o--o--o...--o

Dn:  o--o--o..--o
         |
         o

E6:  o--o--o--o--o
           |
           o

E7:  o--o--o--o--o--o
           |
           o

E8:  o--o--o--o--o--o--o
           |
           o
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In string theory, E8 plays a prominent role as the symmetry group in the theory called the E8 x E8 heterotic string theory, where the 16 dimensions of the E8 x E8 reflection space interpolate between the 26-d bosonic string theory and the 10-d superstring theory. However, E8 xE8 is just one of 5 competing string theory. Recently these five theories have been viewed as different structures within an overarching M-theory, where M stands for “mystery, magic, or membrane, according to taste”--as Edward Witten says. M-theory has as its master structure the 11-d supergravity theory, with E7 as symmetry group. In this way the 7 hidden dimension of the 11-d space time is the 7-d torus, which is the Cartan subgroup of E7. In this way the number 133 (= 7 + 126), as the dimensionality of E7 comes into play. We can add the 4 ordinary dimensions of space time to form the 137-d principal bundle: P(137) --> B(4), with the fiber F(133). So we get 137 the most mysterious number in physics. As Richard Feynman says, “Every good theoretical physicist puts the number 137 up on his wall and wories about it. It should be noted that 11-d supergravity, viewed as a gauge theory, with E7 as gauge group, requires a 137-d principal bundle. It could, of course, be a mere coincidence that the fine structure constant 1/137, i.e. the pure number measure of the strength of the electrical force shows up again in the dimensionality of the E7 principal bundle over ordinary space time.

In any case, 133 as the dimensionality of E7 Lie algebra (and group) is solidly in place as a key part of the structure of M-theory and thus string theory. We are a long way from the treasures of the Templars and their initiation hymn of Psalm 133. Do we not, however, have a greater treasure in the extraordinarily vast object underlying the entire A-D-E hierarchy of mathematical objects?

References:

H.S.M. Coxeter, *Regular Polytopes, 3rd edition,* Dover, New York, 1973.

Michio Kaku, *Introduction to Superstrings and M-Theory, 2nd etition,* Springer-Verlag, New York, 1999. [See especially pp. 462 & 556.]

Saul-Paul Sirag, “Consciousness: A Hyperspace View,” in *Roots of Consciousness,2nd edition,* by Jeffrey Mishlove, Marlow, New York, 1993.

Saul-Paul Sirag, “A Mathematical Strategy for a Theory of Consciousness,” in *Toward a Science of Consciousness: The First Tucson Discussions and Debates,* edited by Stuart R. Hameroff, Afgred W. Kasniak, and Alwyn C. Scott, MIT Press, Cambridge, MA, 1996.